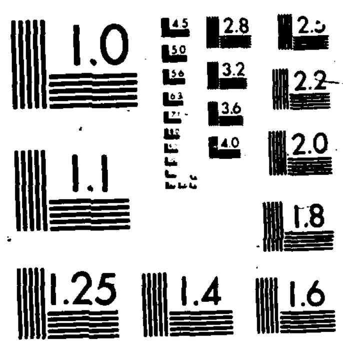
AD-A172 902 MODELING AND ESTIMATION THEORY FOR STOCHASTIC DYNAMICAL SYSTEMS(U) HARVARD UNIV CAMBRIDGE MA DIV OF APPLIED SCIENCES R W BROCKETT 25 SEP 86 ARO-19896.1-MA 1/1

UNCLASSIFIED DAAG29-83-K-0027 F/G 9/3 NL


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

AD-A172 902

MASTER COPY - FOR REPRODUCTION PURPOSES

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARO 19896.1-MA	2. GOVT ACCESSION NO. N/A	3. RECIPIENT'S CATALOG NUMBER N/A
4. TITLE (and Subtitle) Modeling and Estimation Theory for Stochastic Dynamical Systems		5. TYPE OF REPORT & PERIOD COVERED Final Report 01/01/83 -06/30/86
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) R. W. Brockett		8. CONTRACT OR GRANT NUMBER(s) DAAG29-83-K-0027
9. PERFORMING ORGANIZATION NAME AND ADDRESS Division of Applied Sciences Harvard University		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office Post Office Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE 09/25/86
		13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

NA

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

dynamical systems, stochastic systems, estimation theory, control theory, high gain feedback, finite time differential equations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This work investigated the use of pseudorandom models as substitutes for stochastic models in various engineering settings. In particular, certain aspects of estimation theory were developed. The subject of high gain nonlinear feedback was investigated and conditions for all solutions of an autonomous second order system to go to zero in finite time have been derived.

FINAL REPORT

U.S. ARMY RESEARCH GRANT/CONTRACT

DAAG29-83-K-0027

1. Statement of Problem Studied

In this research we investigated the role of random and pseudorandom phenomena in control systems. Our investigations included high gain systems, robotic systems, and computer vision systems. Our work on high gain systems established the first general conditions in the literature which insure that all solutions of a differential equation go to zero in finite time. Moreover, unlike the classical minimum time controllers based on bang-bang ideas, these controllers have continuous (but not Lipshitz continuous) right hand sides. As is now well known, there is a relationship between the Liapunov exponents of an autonomous dynamical system and the dimension of its attractor. In our work we related the Liapunov estimates to the variance of the error in the prediction of the initial conditions. This leads to a satisfying circle of ideas involving the use of pseudorandom models in a prediction theory.

2. Summary of Results

a. Random and Pseudorandom Models

We have investigated the role of pseudorandom models in estimation theory. In particular, we have investigated the extent to which we can estimate the value of a pseudorandom process which is generated by a known model but which is observed in the presence of white noise. We have shown that the characteristic (Liapunov) multipliers of the linearized system are significant in that they determine the linear predictability of the process in the presence of additive noise. This is particularly interesting in view of the Kaplan-Yorke relationship between the dimension of the strange attractor and the Liapunov numbers. It says, in effect, that the linear predictability and the dimension of the attractor are determined by the same aspect of the integral curves and that systems which are more nearly random in that their strange attractor is of a higher dimension are also more difficult to predict as pseudorandom processes. We also carried out a numerical study of the power spectrum of the (apparently) chaotic equation

$$\dddot{y} + \ddot{y} + 1.25\dot{y} + f(y) = 0; \quad f(\cdot) = \text{piecewise linear}$$

and developed some aspects of a theory of estimation which can be applied to pseudorandom problems. The desired results in this study include a deeper understanding of the value of pseudorandom models and useful computation methods for determining the power spectrum.

We also investigated some stochastic modeling and estimation theory problems involving geometry in the plane. In particular we have applied filtering theory to some typical edge detection problems in computer vision. In McIvor's thesis [12] he modeled the edges as planar curves with a Gauss-Markov curvature and formulated a tracking problem. In this way edge tracking becomes a "time" series problem with arc length playing the role of time. He also modeled the curvature process as a Poisson process, ending up with a polygonal fit to the data. This involves adaptation in the sense that edges bifurcate and/or disappear. As part of the formulation of this as a tracking problem, the observation must be modeled. The model chosen was that of an observation corrupted with additive noise. The observation itself is taken to be the two point finite difference approximation to the slope of the edge being tracked. Because the pixel size is significant, this process involves significant quantization error and smoothing is important.

b. <u>High Gain Nonlinear Systems</u>

The problem of deciding what systems can be made linear by the application of feedback was solved some years ago for the class of systems described by

$$\dot{x} = f(x) + g(x) \cdot u$$

with f and g smooth. In many applications f and g are not only nondifferentiable but may be discontinuous. (Static friction is one important source of such a discontinuity.) Linearization by feedback in such cases is not effective because the size and location of the discontinuity may be only approximately known and in such a situation an attempted "exact cancellation" can actually double the size of the discontinuity. A more effective method of dealing with the discontinuity is to superimpose a zero mean high frequency sweep (or "dither") signal on top of the control signal. This results in a system which at low frequencies and small amplitudes is closely approximated by a differential equation with a continuous right hand side. In our work we have set up the problem of approximate linearization as a problem in nonlinear functional analysis and are experimenting with various norms with a view toward selecting those which correspond to desirable control systems behavior. The total least squares method of approximating the graph of a (possibly discontinuous) function is proving to be a very effective tool.

The role of high gain feedback in control systems was examined with a view toward producing control systems which will come to equilibrium in finite time. It is known that high gain can lead to instability and chaotic behavior in many cases -- this research is directed at avoiding these phenomena. For second order equations of the form

$$\ddot{x} + f(x,\dot{x}) = 0$$

the key to choosing f so as to make all solutions go to $y = \dot{y} = 0$ in finite time is to show, first of all, that near zero it must be possible to express $\dot{x}$ as $\dot{x} = \phi(x)$. This means $\ddot{x} = \dot{x}(d\phi/dx) = \phi(x)\phi_x(x) = f(x,\phi(x))$. This latter first order differential equation must then be shown to have a solution passing through $x = 0$. Any such solution has an associated nonuniqueness which complicates the analysis. In Haimo's thesis [10] it is shown how to deal with the nonuniqueness issue and hence with this class of problems.

3. Interactions With Army Personnel

In addition to the normal scientific interaction with army scientists at professional meetings, there were four more intensive opportunities for interaction involving the principal investigator.

a. On January 29, 1984, the principal investigator visited the Night Vision Laboratory at Ft. Belvoir and discussed problems in the area of "smart sensors" with Frank Shields, Vincent Mirelli and others in this group. Partly in response to this visit, we have devoted some effort to picture processing questions.

b. On May 12, 1984, the principal investigator attended the Army Mathematicians Steering Committee meeting in Washington, D.C., and gave a talk on modeling with pseudorandom processes.

c. On May 6-8, 1985, the principal investigator visited army personnel at the Aberdeen Proving Ground and gave a series of lectures describing some current research and recent developments in robotic control.

d. On June 5-6, 1986, the principal investigator visited the army laboratory at Picatinny, N.J., and gave a series of lectures on computer vision and robotics.

4. Publications

[1] Brockett, R. W. and Loncaric, J. "Chaos and Randomness in Dynamical Systems," Proceedings of the 22nd IEEE Conference on Decision and Control. New York: IEEE, 1983. Pp. 1-4.

[2] Brockett, R. W. "Nonlinear Control Theory and Differential Geometry," in Proceedings of the 1982 International Congress of Mathematicians, 1983. Pp. 1357-1367.

[3] Brockett, R. W. "Robotic Manipulations and the Product of Exponentials Formula," in Lecture Notes in Control and Information Sciences, Proceedings of the International Symposium on Mathematical Theory of Networks and Systems. Berlin: Springer-Verlag, 1984. Pp. 120-127.

[4] Brockett, R. W. "Smooth Multimode Control Systems," in Proceedings of the Berkeley-Ames Conference on Nonlinear Problems in Control and Fluid Dynamics. Systems Information and Control, Vol. II (L. Hunt and C. Martin, eds.), Brookline, MA: Math. Sci. Press, 1984. Pp. 103-110.

4. Publications (cont'd)

[5] Brockett, R. W. and Cebuhar, W. "A Prototype Chaotic Differential Equation," in Chaos in Nonlinear Dynamical Systems (Jagdish Chandra, ed.), Philadelphia, PA: SIAM, 1984. Pp. 12-18.

[6] Brockett, R. W. and Loncaric, J. "The Geometry of Compliance Programming," in Theory and Applications of Nonlinear Control Systems (C. I. Byrnes and A. Lindquist, eds.), Amsterdam: Elsevier, 1986. Pp. 35-42.

[7] Loncaric, J. "Normal Forms of Stiffness and Compliance Matrices," (submitted to IEEE Journal of Robotics).

5. Personnel

R. W. Brockett
Wenceslao A. Cebuhar
Douglas W. Cochran
Varda T. Haimo
Morris Lee
Josip Loncaric
Alan M. McIvor
David J. Montana
Stephen R. Peck
Milo B. Sprague
Thomas J. Taylor
Yang Wang

6. Ph.D. Theses by Participating Personnel

[8] Taylor, Thomas J. "Hypoelliptic Diffusions and Nonlinear Control Theory," Ph.D. Thesis, Harvard University, 1983.

[9] Peck, Stephen R. "Combinatorics of Schubert Calculus and Inverse Eigenvalue Problems," Ph.D. Thesis, Harvard University, 1984.

[10] Haimo, Varda T. "Finite Time Differential Equations," Ph.D. Thesis, Harvard University, 1984.

[11] Loncaric, Josip "Geometrical Analysis of Compliant Mechanisms in Robotics," Ph.D. Thesis, Harvard University, 1985.

[12] McIvor, Alan "Stochastically Based Vision Algorithms," Ph.D. Thesis, Harvard University, 1985.

[13] Montana, David J. "Tactile Sensing and the Kinematics of Contact," Ph.D. Thesis, Harvard University, 1986.

END

11-86

DTIC